{ B.Sc Physics (Hons), Part I
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(1)

Q State and Prove Hamilton's principle for conservation system. Give physical significance.

Ans **Statement:-** According to Hamilton's principle "The possible path along which a dynemical system may moves from one point to another in the configration space withen a given interval of time, the actual path followed is that for which the time integral of the lagrangian function for the system is an extremum."

Mathematically Hamilton principle state that the motion of the system from time t_1 to t_2 such that the line integral

$$\int_{t_1}^{t_2} L\,dt = I \quad \text{------} \quad (1)$$

where $L = T - V$ is an extremum for time path of motion.

Here I is the extremum value of the time integral of the lagrangian and is known as Hamilton's principle for the path eqn (1) may be represents as in terms of the ~~calculas~~ calculus of variation

$$\delta \int_{t_1}^{t_2} L\,dt = 0$$

Deduction:→ Let us consider that the conservative ~~holonmic~~ holonomic dynamical system moves from P to Q which are initial and final confugration of the system at time t_1 and t_2.

Let PQR be the actual path and PR′Q and PR″Q be the two neighbouring paths out of infine nos of possibilities.

For the derivation of H.P, the following assumptions are satisfied.

(1) δt must be equal to zero at the end point
(2) δt should be equal to zero at the end point i,e the ~~p~~ point 'P' and 'Q' are fixed in space.

Let us consider the system be acted upon by no of forces represented by F. Let the ith particle acted upon force F_i acquire acceleration $\ddot{r}_i$, so that

$$F_i = m_i \ddot{r}_i$$

from D' Alemberts eq particle, we get

$$\Sigma(F_i - m_i \ddot{r}_i)\cdot \delta r_i = 0$$

$$\Sigma(F_i \delta r_i - m_i \ddot{r}_i \delta r_i) = 0$$

$$\Rightarrow \ddot{r}_i \cdot \delta r_i = \frac{d}{dt}(\dot{r}\cdot \delta r_i) - \dot{r}_i \frac{d}{dt}(\delta r_i) \quad - - - - ②$$

If there is little variation in actual and neighbouring paths we have $\delta r_i = r_i' - r_i$ (Let

$$\frac{d}{dt}(\delta r_i) = \frac{d}{dt}(r_i' - r_i) = \frac{dr_i'}{dt} - \frac{dr_i}{dt} = \delta\left\{\frac{dr_i}{dt}\right\} = d\dot{r}_i$$

from eq^n ②

$$\ddot{r}_i - \delta r_i = \frac{d}{dt}(\dot{r}_i \cdot \delta r_i) - \dot{r}_i \cdot \delta(\dot{r}_i)$$

from eq^n ① $\sum_i F_i \delta r_i - \Sigma m_i \left[\frac{d}{dt}(\dot{r}_i - \delta r_i) - \dot{r}_i \cdot \delta(\dot{r}_i)\right] = 0$

$$\Rightarrow \sum_i F_i \delta r_i - \sum_i m_i \left[\frac{d}{dt}(\dot{r}_i \cdot \delta r_i) - \delta \frac{1}{2}(\dot{r}_i)^2\right] = 0$$

$$\Rightarrow \sum_i F_i \delta r_i + \Sigma \frac{1}{2} m_i (\dot{r}_i)^2 = \Sigma m_i \frac{d}{dt}(\dot{r}_i \cdot \delta r_i)$$

In the above equation that $\Sigma \frac{1}{2} m_i v_i^2$ is the K.E (T) of the ~~system~~ system of the particle and if the forces are conservative $\Sigma F_i \delta r_i = -\delta V$. Here V is P.E. Then the above eq^n becomes

$$\{\delta T - \delta V\} = \frac{d}{dt}\left[\sum_i (m_i \dot{r}_i) d r_i\right]$$

on integrating from t_1 to t_2,

$$\int_{t_1}^{t_2} \delta(T-V)\, dt = \int_{t_1}^{t_2} \frac{d}{dt} \sum_i (m_i \dot{r}_i)\, \delta r_i\, dt$$

$$\Rightarrow \delta \int_{t_1}^{t_2} (T-V)\, dt = \sum_i \left[(m_i \cdot \dot{r}_i)\, \delta r_i\right]_{t_1}^{t_2} = \sum_i \left[(m_i \cdot \dot{r}_i)\, \delta r_i\right]_P^Q = 0$$

Since δr_i = out of end ~~path~~ point

or, $$\delta \int_{t_1}^{t_2} L\, dt = 0$$

$$\therefore \int_{t_1}^{t_2} L\,dt = I = \text{extremum}$$

This is the Hamilton's prenceple.

Physical Significance :-

Eqn ① represents that the integral of the variation of the K.E plus virtual work done during the virtual displacement from actual to ~~Virtual path~~ varied path must be zero which is extended Hamilton's prenceple.